Usefulness of Collision Warning Inter-Vehicular System

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Abstract

This paper studies the contribution of *Inter-Vehicular communications (IVC)* in a vehicle string. We suppose that all, or, a subset of the vehicles are equipped with radio devices, enabling communication between them. We evaluate the benefit of the dissemination of warning messages for the reduction of accidents. More precisely, we evaluate and compare the number of vehicles involved in a multiple-crash caused by an accident, with and without the use of *IVC*. These evaluations are made under different scenarios, from simplistic to more realistic assumptions. We derive analytical formulae for most of the cases, and use simulations for the complicated ones. We show that even when a poor proportion of vehicles use *IVC*, the number of collisions is drastically reduced.

KEYWORDS: inter-vehicular communication, safety communications, collision warning message, system penetration rate, rear-end collision

1 Introduction

In recent years, *Inter-Vehicle Communication (IVC)* has became an intensive research area, as part of Intelligent Transportation Systems. It supposes that all, or, a subset of the vehicles are equipped with radio devices, enabling communication between them. These communications usually use the ad hoc mode. This allows a vehicle to communicate directly with another vehicle without the use of any dedicated infrastructure (Base Station, Access Point, etc.). Although classical 802.11 can be used for IVC, specific technologies like IEEE 802.11p [6] (also referred to as Wireless Access in Vehicular Environments, *WAVE*) shows a great deal of promise. This standard scheduled to be published in 2009, includes data exchanges between vehicles and between infrastructure and vehicles with a greater radio range than classical 802.11. Also, by using ad hoc mode of these radio technologies (all the 802.11 technologies have an ad hoc mode), we gain the advantage that the scope of the communications is not just limited to the radio range. Here, the vehicles can act as routers, i.e. they implement forwarding and routing algorithms, thus they form a Multi-Hop wireless Ad-hoc NETwork (MANET) which ensures a good dissemination of the messages.

One of the major applications of *IVC*, the one considered in this paper, is the dissemination of warning information [7]. This allows a vehicle to obtain information about accidents from other vehicles. Such applications rely on broadcast algorithms [1, 3, 4, 12]. These algorithms are entrusted the task of disseminating warning messages quickly and efficiently through the network. The radio scope of the communication devices, up to 1 km for IEEE 802.11*p*, ensures the connectivity of the network, and guarantee that the warning messages can be received at several kilometer distances from the sender. Moreover, most of the proposed broadcast algorithms disseminate information in a few hundreds of milliseconds in the whole network. For instance, the algorithm presented in [12] delivers broadcast messages in less than 100ms to nodes that are 10 kilometers away. Furthermore transmitting a one hop message (up to 1km away) takes an average time of less than 1ms. These performances ensure a quasi-instantaneous transmission of warning messages, in the vicinity of the accident, to the end user (the driver) when compared with its higher reaction time (1 second [13]).

Warning messages are essentiel for road safety because they allow the vehicle to react to indirectly detectable events. Although some works have been concerned with the study of collision on the road [2, 5], little research effort has been devoted to study the benefit of warning communications in collision reduction. [11] examines the reduction of the Average Accident Interval (AAI) by means of communication. It concludes that a high diffusion ratio of communication system (>60%) is necessary in order to increase significantly the AAI. Further works concerning the joint study of communications and sensors lead to similar results [10]. [8] shows that warning communications allow an enhancement of the safetycapacity relation thus indicating a reduction of the collisions for a constant road capacity. [9] evaluate the impact of communication when considering the use of various sensors in a classical pile up scenario with the front car breaking (but without initial collision of the front car). It indicates that with a 50% penetration, almost all heavy collisions are excluded. A similar scenario has been considered in this paper but we assume an initial collision that activate a minimal warning communication system (the system sends only the warning message: no add-on localization or sensing messages are sent). The aim of this paper is to study the penetration ratio which is necessary to drastically reduce the secondary collisions.

In the Section II, we present the early warning communication problem and introduce the notations and the capacity concept. Section III introduces two analytical formula describing the number of collision in a fully or none equipped string of vehicles. Then section IV extends this works to a string of partially equipped vehicles and Section V presents simulation results. We conclude in Section VI.

2 Problem Statement

In order to analyze the usefulness of inter-vehicular warning communications, we consider a string of vehicles whose leader has crashed with a stationary, heavy obstacle (e.g. a bridge that has just collapsed, a truck that is stopped...). The other drivers (human or computer) of the vehicles brake as soon as they are aware of the situation. They can be aware due to their own sensors or from a message broadcasted by the crashed vehicle. This section presents the notations and the capacity notion so as to study the impact of warning communications (after an initial collision) on the number of collisions.



Figure 1: Emergency braking

2.1 Notations and assumptions

Let us consider a sub-string of two vehicles (Veh_i and Veh_{i+1}) of a vehicle string (Fig. 1). Veh_i is the leader vehicle of this sub-string and Veh_{i+1} is the follower. Each vehicle Veh_i is characterized by the following parameters:

- v_i , the velocity of the i^{th} vehicle (in m/s),
- γ_i , the absolute value of breaking capacity of the *i*th vehicle (in m/s²),
- x_i^- : the position of the rear of the i^{th} vehicle (in m),
- x_i^+ : the position of the front of the i^{th} vehicle (in m),
- l_i , the length of the i^{th} vehicle (in m), with $l_i = x_i^+ x_i^-$
- *d_{inter(i,i+1)}*, the interdistance between the *ith* and the (*i*+1)th vehicle when they are in motion (in m),
- τ_i the reaction time of the driver (in s),
- $d_{\tau_i} = \tau_i v_i$, the distance covered during the reaction time τ_i (in m),
- d_{dec_i} , the deceleration distance of the i^{th} vehicle (in m),
- $d_{stop_i} = d_{\tau_i} + d_{dec_i}$, the stop distance (which includes the deceleration distance) of the i^{th} vehicle (in m),
- $\varepsilon_{i,i+1}$ the remaining interdistance between the i^{th} and the $(i+1)^{th}$ vehicles when they become stationary (in m). This value is also called security offset.

We denote initial conditions by a zero exponent, thus: $x_i^{-,0}$, $x_i^{+,0}$ and v_i^0 are the initial values respectively of, the rear position, the front position and the velocity of the *i*th vehicle. The initial moment is the moment when the perturbation occurs (the first vehicle collides).

To simplify the analysis, we assume the vehicle string to be homogeneous (i.e. all the vehicles have the same characteristics) and when two vehicles collide, the length of the formed agglomerate is 2l (no compression). Also, communication is considered ideal, i.e. latency and jitter are assumed null. There is no multi-path problem and the signal range is infinite. Thus, when a vehicle emits a warning message, all other vehicles are informed instantaneously.

2.2 Capacity

Just before the perturbation, the density (the number of vehicles / length) ρ is defined as:

$$\rho = \frac{1}{\bar{l} + \bar{d}_{inter}^0} \tag{1}$$

where \bar{l} is the mean length of the vehicles and \bar{d}_{inter}^0 the averaged initial interdistance (when the string is homogenous $\bar{l} = l$ and $\bar{d}_{inter}^0 = d_{inter}^0$). From this spacial repartition, we can define a temporal repartition of the vehicles as:

$$c = v\rho = \frac{v}{\bar{l} + \bar{d}_{inter}^0} \tag{2}$$

where c is the capacity of the vehicles flow (number of vehicles/time)[6] and v is the velocity of the flow.

3 Strings of unequipped or fully equipped vehicles

We give in this section an analytical formulation of the number of collisions in a homogeneous string of unequipped vehicles. Next we present another analytical forumula when all the vehicle use inter-vehicular communication.

3.1 Vehicle String Without Warning Communications

Without warning communications, the driver (a human or a computer) of the vehicle Veh_{i+1} brakes after seeing the brake lights of the vehicle Veh_i . The time of the overall process, starting breaking and seeing the brake lights, is the reaction time τ . The vehicle Veh_1 starts braking (after a reaction time τ) when its driver sees Veh_0 colliding with the huge obstacle.

As the vehicle string is homogeneous, when the front of the first's vehicle Veh_0 collides with the obstacle at time $t_{collision}$, the i^{th} vehicle is $(l + d_{inter}).i$ meters far from the obstacle. The braking of each vehicle is delayed by the reaction time for each vehicle since each vehicle has a vision limited to its front vehicle. Thus, when Veh_i starts to break, $i\tau$ seconds have already passed after the initial collision with the obstacle. The reaction time τ effect is a cumulative effect. The stop distance of Veh_i is $d_{stop} = i.d_{\tau} + d_{dec}$ meters. When the first i - 1 vehicles have collided, the agglomerate is i.l meters long. Therefore, Veh_i has to be at least $d_{stop} + i.l$ meters far from the obstacle at the time $t_{collision}$ to avoid collision with the $(i - 1)^{th}$ vehicle:

$$x_i^{+,0} \ge i.d_\tau + d_{dec} + i.l \tag{3}$$

which can be rewritten as:

$$(l+d_{inter}).i \ge i.d_{\tau} + d_{dec} + i.l \tag{4}$$

The vehicle whose number is bigger or equal to *i* will not collide:

$$i \ge \frac{d_{dec}}{d_{inter} - d_{\tau}} \tag{5}$$

And the number of collided vehicles is:

$$C = \left\lfloor \frac{d_{dec}}{d_{inter} - d_{\tau}} \right\rfloor \tag{6}$$

where $\lfloor \rfloor$ means the integer part of the fraction. The number of collision is maximal (infinite if we consider an infinite string) when $d_{inter} = d_{\tau}$: the vehicles have not enough time to decelerate before colliding with their front vehicle.

3.2 Vehicle String With an Ideal Communication Technology

With an ideal communication technology, all other vehicles are informed when the first vehicle collides with the ostacle. This scenario corresponds to a case where drivers can see the brakes lights of all the vehicles which are ahead of them. When a driver is alerted, he brakes after his reaction time τ . Compared to the previous case (without warning communication), here we notice that τ is not cumulative anymore. Thanks to communications, reaction times appear as concurrent operation time. The equation (5) becomes:

$$(l+d_{inter}).i \ge d_{\tau} + d_{dec} + i.l \tag{7}$$

because the drivers break after d_{τ} (rather than *i.d*_{τ}, see Eq. 4). Therefore, the number of collided vehicles is:

$$C = \left\lfloor \frac{d_{dec} + d_{\tau}}{d_{inter}} \right\rfloor \tag{8}$$

The number of collided vehicles is finite as soon as we consider $d_{inter} \neq 0$.

4 String of partially equipped vehicles

In this Section, we extend the previous model in order to consider a string of vehicles partially equipped with communication devices. We consider two models. In the first, the distances between the vehicles are supposed to be constant. For the second model the interdistances are distributed based on a statistical distribution model.

4.1 The equipment dissemination and its effect on the reaction time

We assume that a vehicle is equipped with a radio with probability p, independent of the other vehicles. The leading vehicle (vehicle 0) is equipped¹ and it emits instantaneously a warning message to all other equipped vehicles when it collides. Thus, all vehicles which receive this warning brake after their reaction time τ . The other vehicles, not equipped with a radio, brake at a time τ after the vehicle in front of them brakes.

More formally, let us consider the i^{th} vehicle, and let X_i be its associated random variable. X_i describes the index of an equipped vehicle which is the nearest vehicle to the i^{th} vehicle (from the leading vehicle). The nearest equipped vehicle can be the ego vehicle. X_i brakes after a reaction time τ takes its values in $\{1, 2, ..., i\}$. Consequently, Veh_i will brake after $(i + 1 - X_i)\tau$ seconds (this value is still avalable when there is none equipped vehicle by considering $X_i = 1$).

4.2 Model with Constant Interdistance

We suppose that a vehicle is equipped with a probability p independent of other vehicles and X_i has the following distribution:

$$\begin{cases} \mathbb{P}(X_i = 1) = (1 - p)^{i - 1} \\ \mathbb{P}(X_i = k) = p(1 - p)^{i - k} \text{ for } k \in \{2, ..., i\} \end{cases}$$
(9)

Indeed, $X_i = 1$ if the i - 1 leading vehicles do not have a radio, and $X_i = k$ if the i - k leading vehicles do not have a radio whereas the i^{th} has one. Let $Z_i = i + 1 - X_i$, the number of vehicles in the string of non equipped vehicle starting from the i^{th} vehicle (to the leading vehicle), taking its values in the set $\{1, ..., i\}$. From the distribution of X_i , the computation of Z_i distribution is straightforward:

$$\begin{cases} \mathbb{P}(Z_i = k') = p(1-p)^{k'-1} \text{ for } k' \in \{1, .., i-1\} \\ \mathbb{P}(Z_i = i) = (1-p)^{i-1} \end{cases}$$
(10)

Veh_i will crash if it cannot stop before its preceding vehicle:

$$crash(i) = \begin{cases} 1 & \text{if } Z_i d_\tau + d_{dec} > i.d_{inter} \\ 0 & \text{otherwise} \end{cases}$$
(11)

Let *C* be the random variable describing the number of collisions between the vehicles, *C* can be defined as:

$$C = \sum_{i=1}^{+\infty} crash(i) \tag{12}$$

We are interested in the computation of the mean value of *C*, denoted as $\mathbb{E}[C]$. The esperance of the sum is the sum of the esperance (as the conditions of Fubini's theorem are verified

¹If the vehicle 0 is not equipped then the study "without warning communication" (see Section 3.1) is available for the first part of the string. As soon as an equipped vehicle collide, wa can consider this vehicle as vehicle 0 (the leading vehicle in this study).

when $d_{inter} > d_{\tau}$). The esperance of the condition given by equation (11) is the probabibility that the condition holds. We get,

$$\mathbb{E}[C] = \mathbb{E}\left[\sum_{i=1}^{+\infty} crash(i)\right] = \sum_{i=1}^{+\infty} \mathbb{P}(Z_i d_\tau + d_{dec} > i.d_{inter})$$
(13)

$$= \sum_{i=1}^{+\infty} \mathbb{P}\left(Z_i > \frac{i.d_{inter} - d_{dec}}{d_{\tau}}\right)$$
(14)

$$\begin{split} \text{If } 1 &\leq \frac{i.d_{inter} - d_{dec}}{d_{\tau}} \leq i \text{ then} \\ \mathbb{P}\left(Z_i > \frac{i.d_{inter} - d_{dec}}{d_{\tau}}\right) &= \sum_{j=\left\lceil \frac{i.d_{inter} - d_{dec}}{d_{\tau}} \right\rceil}^{i-1} p \left(1 - p\right)^{j-1} + (1 - p)^{i-1} = (1 - p)^{\left\lceil \frac{i.d_{inter} - d_{dec}}{d_{\tau}} - 1 \right\rceil} \end{split}$$

Thus,

$$\mathbb{P}\left(Z_{i} > \frac{i.d_{inter} - d_{dec}}{d_{\tau}}\right) = \begin{cases} 1 & \text{if } \frac{i.d_{inter} - d_{dec}}{d_{\tau}} < 1\\ (1-p)^{\left\lceil \frac{i.d_{inter} - d_{dec}}{d_{\tau}} - 1 \right\rceil} & \text{if } 1 \le \frac{i.d_{inter} - d_{dec}}{d_{\tau}} \le i \\ 0 & \text{if } \frac{i.d_{inter} - d_{dec}}{d_{\tau}} > i \end{cases}$$
(15)

Finally, the mean number of collision can be computed thanks to Eq. (14) and Eq. (15).

4.3 Model with Randomly Distributed Interdistances

When the interdistance between vehicles are randomly distributed, the i^{th} vehicle will crash if

$$\sum_{k=1}^{l} d_{inter(k-1,k)} < (i - X_i + 1)d_{\tau} + d_{dec}$$
(16)

leading to

$$\sum_{k=1}^{i} d_{inter(k-1,k)} < Z_i d_{\tau} + d_{dec}$$
(17)

By applying the methodology followed in the previous subsection, we obtain

$$\mathbb{E}\left[C\right] = \sum_{i=1}^{+\infty} \mathbb{P}\left(\sum_{k=1}^{i} d_{inter(k-1,k)} < Z_i d_{\tau} + d_{dec}\right)$$

If we consider that the random variables $(d_{inter(i-1,i)})_{i\geq 1}$ are independently and exponentially distributed, $\sum_{k=1}^{i} d_{inter(k-1,k)}$ follows a *Gamma* distribution with parameters $(i, \frac{1}{d_{inter}})$, where \bar{d}_{inter} is the mean interdistance. The average number of collisions can thus be expressed with regard to the *cumulative distribution function* of the *Gamma* distribution (denoted $F_{\Gamma(.,.)}$) by:

$$\mathbb{E}[C] = \sum_{i=1}^{+\infty} \left[\sum_{k=1}^{i-1} F_{\Gamma(i,d_{inter})} \left(k \cdot d_{\tau} + d_{dec} \right) p(1-p)^{k-1} + F_{\Gamma(i,d)} \left(i \cdot d_{\tau} + d_{dec} \right) (1-p)^{i-1} \right]$$
(18)

5 Simulation Results

5.1 Basic Numerical Application

Parameters	Numerical Values
Vehicle velocity	$v_i = 36.1 \text{ m/s} (130 \text{ km/h})$
Vehicle length	l = 5 meters
Capacity	$c \in [1800, 3200]$ Veh/h
Mean intervehicle distance	$d_{inter} = \frac{v}{c/3600} - l$
Vehicle braking capacity	$\gamma = 0.8g$
Reaction time	$\tau = 1$ second
Distance covered during $ au$	$d_{\tau} = v\tau = 36.1 \text{ m/s}$

Table 1: Numerical Values of Simulation parameters



Figure 2: Mean number of collisions for a string of vehicles and for different proportion of vehicles equipped with *IVC* devices.

For the numerical evaluation, we use the set of parameters given in table 5.1. The range of the capacity c has been chosen in such a way that $d_{\tau} < d_{inter}$. It leads to the upper value of c: c = 3200 veh/h. For greater values of c, the number of collisions is infinite. The lower value c = 1800 veh/h, corresponds to a small capacity of vehicles. For this value, there is a distance of approximately 70 meters between two successive vehicles. In Figure 2(a), we show the mean number of collisions for different penetration ratio p of communication technology and for constant distances between the vehicles. This curve is obtained from formula (??).

The difference between the number of collisions with and without *IVC* is small for low values of the capacity c. For example, when c = 2000 veh/h (the inter-distance is approximately 60 meters), there are 2 collisions on average without *IVC*, and approximately 1.99 for p = 1% and 1.95 for p = 5%. The first notable improvement is for p = 25%. This small difference is explained by the facts that the number of collisions is small, and that collisions are inevitable for the first few vehicles just behind the crashed one. However,



Figure 3: Mean number of collisions for a string of vehicles as function of the penetration ratio *p*.

when the traffic is dense, it is clear that the use of *IVC* decreases the number of collisions. Even for small values of p, there is significant reduction of collisions. When c = 3050 (interdistance is equal to 37.6 meters) corresponding to the edge of the plot, the number of collisions are reduced from 55 to 42 for p = 1% only, and to 20 for p = 5%. For a sufficient penetration ratio, approximately 25%, there is an obvious benefit in using *IVC* as the number of collisions stay very small even for larger values of the capacity.

In Figure 2(b), we plot the mean number of collisions, given by equation (18), for exponentially distributed intervehicle distances. Observations are roughly the same as Figure 2(a): i.e. communication decreases the number of collisions whatever the penetration ratio p, but collisions are really reduced for $p \approx 25\%$. It is worth noting that the number of collisions are greater when the interdistances are exponentially distributed.

It seems, that *IVC* has the greatest effects when p varies between 5% and 25%. In Figure 5.1, we plot the number of collisions as function of p for different values of c (c = 1800, 2200, 2600 and 3000), and when the interdistance is constant and follows an exponential distribution. As expected, for large values of c, the number of collisions decreases very quickly for 0 . For <math>p greater than 25%, the number of collisions stabilizes and stay more or less constant. Thus, it suffices to have a penetration ratio of approximately 25% to keep the number of collisions very low.

5.2 Numerical method for other distributions

As the *cdf* of $\left(\sum_{k=1}^{i} d_{inter_{k-1,k}}\right)$ in equation (18) is difficult to compute for other general distributions of $d_{inter_{k-1,k}}$, we use a numerical method to take them into account. We simulate a chain of vehicles with different interdistance distributions.

To obtain a sample of the number of collisions, we begin by generating a chain of vehicles. The interdistance between two successive vehicles is randomly drawn according to the considered distribution. A vehicle is then equipped with a radio device with probability



(a) Intervehicle distances follow Normal distributions (b) Intervehicle distances follow Normal distributions with $\sigma = \frac{d}{2}$. with $\sigma = \frac{d}{4}$.

Figure 4: Mean number of collisions for a string of vehicles and for different proportion of vehicles equipped with *IVC* devices.



Figure 5: Comparisons of the mean number of collisions for the different distributions, and for p = 1,5 and 25%.

p independent of the other vehicles. The number of collisions for this configuration is then simulated. The mean number of collisions is obtained as the average of 1000 samples.

The distribution chosen to generate interdistance between the vehicles is the Normal distribution $N(d_{inter}, \sigma^2)$ with mean d_{inter} and variance σ^2 . In the simulations, d_{inter} is computed in such a way that it fits the value of c. In other words, d_{inter} is computed from formula (2). In order to avoid negative interdistances, the Normal distribution is truncated (we neglect the negative samples). As it introduces an asymetry in the distribution, and a shift of the mean value, we truncate the Normal distribution on both sides. The considered probability density function is then

$$f_{N(d_{inter},\sigma^2)|_{[0,2d_{inter}]}}(x) = Ae^{-\frac{(x-d_{inter})}{2\sigma^2}} \mathbb{1}_{[0,2d_{inter}]}(x)$$

Where $\mathbb{1}_{Condition}$ is the indicator function which equals to 1 if *Condition* is true and 0 otherwise, and *A* is a normalizing factor. For the simulation results shown in Figures 4(a) and 4(b), we use the set of parameters of table 5.1 and the truncated Normal distribution to generate the interdistance. We choose two different values of σ ($\sigma = \frac{d}{2}$ and $\frac{d}{4}$) to evaluate the impact of variance on the number of collisions. The curves of the two figures present the same behaviors, but for the same value of *p*, the number of collisions is slightly greater when $\sigma = \frac{d}{2}$.

We also plot in Figure 5.2, the number of collisions for different distributions. It allows us to measure the impact of distributions on collisions. For small values of p (p = 1%), there is a significant differences between the distributions. For example, when p = 1% and c = 2800, the number of collisions varies from 12 to 26 with regard to the four distributions. It shows that the different interdistance distributions with same means, may lead to very different results. However, when p is approximately greater or equal than 20%, the difference between the four distributions is negligible. The effects of the distributions, in particular their variance (equal to d_{inter}^2 for the exponential law, approximately $\frac{d_{inter}^2}{2}$ and $\frac{d_{inter}^2}{4}$ for the truncated Normal distributions and 0 for the constant interdistance) does not impact at all the performance for sufficiently large p.

We also considered the exponential distribution in our simulations. The results have been used to validate the theoritical results of Section 4 and are not shown here.

6 Conclusion

This article focused on the impact of *Inter-Vehicle Commication (IVC)* on the number of collisions involved in a multi-crash accident. We derived analytical formulae for the number of collisions for both constant and random inter-vehicle distances. For some special distributions we use simulations in order to augment the theoritical results. We assume that only a proportion p of vehicles is equipped with radio devices. We discussed the impact of p on the number of collisions. The different results show that communication considerably reduces the number of collision, even with a low dissemination. For a large enough p, approximately 25%, the number of collisions stay constant with the mean number of vehicles per hour. It demonstrates that a partial deployement of *IVC* is sufficient to reduce drastically the number of collisions.

Further works will concern with the impact of IVC in the reduction of the severity of the

collisions. Studying the impact of communication with the number of collision compared with the severity of collisions can lead to radically different results. For instance, it is more advisable to manage a collision mitigation i.e. to have several weak collisions (where no body is injured) than a huge one (where people are injured). Our future papers will adress this topic.

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