An analysis of the MPR selection in OLSR

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Abstract— **OLSR is a recent routing protocol for multi-hop wireless ad-hoc networks standardized by the IETF. It uses the concept of Multi-Point Relay to minimize the overhead of routing messages and limit the harmful effects of broadcast in such networks. In this paper, we are interested in the performance evaluation of Multi-Point Relay selection. We analyze the mean number of selected MPR in the network and their spatial distribution.**

I. INTRODUCTION

Due to the emergence of wireless local area network technologies such as 802.11 [3], hyperlan [4] or bluetooth [5], the use of mobile wireless networks is growing fast. With these technologies, new challenges arise such as connecting wireless nodes without any infrastructure. In order to connect nodes which are not in each other's radio range, packets need to be relayed by intermediate nodes. Such networks thus require forwarding capabilities in the nodes and a routing protocol to find the available path to the destination. Routing in such a wireless environment is very different to classical routing in wired networks. Indeed, nodes are mobile by essence and may vanish or appear due to the wireless nature of the physical layer. The topology is thus in constant evolution. However, routing advertisements are expensive in resources since a node spends energy while transmitting as well as receiving and each message sent by a node is received systematically by all its neighbors. Therefore, broadcasted advertisements must be limited in order to maximize the network lifetime. Consequently, an accurate routing protocol needs to be distributed, must guarantee a low level of traffic control overhead but should be able to rapidly take into account link failures due for instance to node movements. The Internet Engineering Task Force (IETF) addresses the design of such protocols in its MANET[1] (Mobile Ad-hoc Networks) working group.

One of the recent standardized protocols is OLSR (Optimized Link State Routing Algorithm) presented in [1], [2]. In OLSR, only a subset of preselected nodes called MPR (Multi-Point Relay) are used to perform topological advertisements. At the same time, control messages (containing *e.g.* routing information) are broadcasted and forwarded only by these MPR. Overhead is thus minimized and the well known storm problem [6] due to broadcast is avoided.

In this paper, we are interested in the performances of MPR selection. We analyze the mean number of selected MPR in the network and their spatial distribution. We then show that the algorithm used for selecting the MPR is very efficient and that the different proposed variants of the algorithm always lead to very close performances.

In Section II, we detail the OLSR protocol and the MPR selection algorithm. In Section III, we give results about probabilities and mean quantities relative to the MPR selection algorithm. Numerical results and simulations are presented in Section IV. We conclude and discuss future works in Section V.

II. OLSR

OLSR is a proactive routing protocol for Mobile Ad-hoc Networks (MANET), *i.e.*, a network topology is permanently updated on each node in order to provide a route as soon as needed. It uses the concept of Multi-Point Relay to minimize the overhead of control traffic and to provide shortest path routes (in number of hops) for all destinations in the network. Each node chooses in its neighborhood a subset of nodes: its MPR. A MPR set is thus relative to each node. Each node keeps the list of its neighbors which have selected it as MPR. This list is called the MPR-selector list. It is obtained from HELLO packets which are periodically sent between neighbors. In order to build the database to route the packets, all the MPR broadcast their MPR-selectors in the network. The shortest path to all possible destinations is then computed from these lists, a path between two nodes being a sequence of MPR.

Since only MPR are authorized to send their MPR-selectors, the control traffic is drastically reduced compared to classical link-state algorithms. The MPR are

[1] http://www.ietf.org/html.charters/manet-charter.html

also used to minimize the flooding of broadcast messages, as only MPR transmit them. When receiving a broadcast message M from a node u, a node v forwards it only if it is the first time v receives M and if node u is in node v's MPR-selectors list. This technique allows to reduce the number of transmitters of broadcasted messages. In the next section, we detail the algorithm which allows a node to select its MPR within its neighborhood. It consists of choosing nodes such that the whole 2-neighborhood is covered.

A. MPR selection

As the optimal MPR selection is NP-complet ([10]), we give here the Simple Greedy MPR Heuristic which is the one currently used in the OLSR implementation.

For a node u, let $N(u)$ be the neighborhood of u. $N(u)$ is the set of nodes which are in u's range and share a bidirectional link with u. We denote by $N_2(u)$ the 2-neighborhood of u, *i.e*, the set of nodes which are neighbors of at least one node of $N(u)$ but which do not belong to $N(u)$. ($N_2(u) = \{v \text{ s.t. } \exists\, w \in N(u) \,|\, v \in N(w) \setminus \{u\} \cup N(u)\}$). For a node $v \in N(u)$, let $d_u^+(v)$ be the number of nodes of $N_2(u)$ which are in $N(v)$:

$$d_u^+(v) = |N_2(u) \cap N(v)|$$

For a node $v \in N_2(u)$, let $d_u^-(v)$ be the number of nodes of $N(u)$ which are in $N(v)$:

$$d_u^-(v) = |N(u) \cap N(v)|$$

The node u selects in $N(u)$, a set of nodes which covers $N_2(u)$. We define as $MPR(u)$ this set of MPR selected by u. In other words, $MPR(u)$ is such that:

$$\bigcup_{v \in MPR(u)} N(v) = u \cup N(u) \cup N_2(u)$$

The algorithm is the following:

Algorithm 1 Simple Greedy MPR Heuristic

```
For all node u ∈ V
    For all node v ∈ N(u)
        if (∃ w ∈ N(v)∩N2(u) | d_u^-(w) = 1) then
            Select v as MPR(u).
            ▷ Select as MPR(u), nodes for which there is a
            node of N2(u) which has v as single parent in
            N(u).
            Remove v from N(u) and remove N(v) ∩
            N2(u) from N2(u).
        end
    while (N2(u) ≠ ∅)
        For all node v ∈ N(u)
            if (d_u^+(v) = max_{w∈N(u)} d_u^+(w)) then
                Select v as MPR(u).
                ▷ Select as MPR(u) the node v which cover
                the maximal number of nodes in N2(u).
                Remove v from N(u) and remove N(v)∩
                N2(u) from N2(u).
            end
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The first step selects as MPR the nodes which cover "isolated nodes of $N_2(u)$". The nodes covered this way have a single neighbor in $N(u)$ and thus must be included into the set of MPR if we want to cover the whole 2-neighborhood. Therefore, only the second step of the algorithm can be improved in order, for instance, to find the minimum number of MPR.

B. Related works

Most of the literature about the performances of OLSR deals with the efficiency of the OLSR routing protocol itself or the different flooding techniques using MPR ([7], [8], [9], [10]). Only a few papers have studied the different algorithm performances for the MPR selection. An analysis of the MPR selection on the line is given in [13]. Other analytical results in random graphs and random unit graphs are also given in [8]. For instance, a rough upper bound on the size of the MPR set is given in a random unit graph. Other very interesting results are presented in [12]. The authors have proposed and analyzed other heuristics for selecting MPR. Alternative algorithms to the classical MPR algorithm described above are given in order to reduce the number of collisions, minimize the overlap between MPR or maximize the global bandwidth. All results for the proposed algorithms are quite similar, particularly for the mean number of MPR per node. Indeed, as we will show, almost all the MPR of a node are selected during the first step of the algorithms. This first step cannot be changed as it consists of choosing neighbors which cover isolated nodes (nodes in the 2-neighborhood covered by a single neighbor). Therefore, these nodes must be chosen (in order to cover the whole 2-neighborhood) and they must be chosen first in order to minimize the number of MPR.

III. Analysis

We are interested in the properties of the MPR set of a typical node. Therefore, we do not consider the whole network but only a "typical point" located at the origin of the plane and its 1 and 2-neighborhood. Our model is similar to the classical unit random graph used to model ad-hoc networks. This family of models is not completely realistic since it omits interference between the nodes. More realistic models have been proposed, for instance in [14] where the authors present an accurate model for a CDMA network. However, we have chosen a more general model since we do not make any assumptions about the wireless technology used by the nodes.

Let be a Poisson point process on $B(0,2R)$ of intensity $\lambda > 0$. We add a point 0 at the origin for which we study the MPR selection algorithm. We assume that there is a bidirectional link between two nodes if and only if $d(u,v) \leq R$ where $d(u,v)$ is the Euclidean distance between u and v and $R \in I\!R^{+*}$ a constant. The neighborhood of the point 0 is thus constituted of the points of the Poisson point process which are in $B(0,R)$. We use the notation already defined in Section II: N (resp. N_2) is the 1-neighborhood (resp. the 2-neighborhood) of the point 0.

A. General results

Let's note $A(r)$ the area of the intersection of two balls of radius R and where the distance between the centers of the balls is r and $A_1(u,r,R)$ the area of the union of two discs of radius R and u and where the centers are distant from r:

$$A(r) = 2R^2 \arccos\left(\frac{r}{2R}\right) - r\sqrt{R^2 - \frac{r^2}{4}}$$

and

$$\begin{aligned} A_1(u,r,R) &= rR\sqrt{1 - \frac{R^2 - u^2 + r^2}{2Rr}} \\ &+R^2\left(\pi - \arccos\frac{R^2 - u^2 + r^2}{2Rr}\right) \\ &+u^2\left(\pi - \arccos\frac{r}{2u}\left(1 - \frac{R^2 - u^2}{r^2}\right)\right) \end{aligned}$$

***Proposition** 1:* Let u be a point uniformly distributed in $B(0,R)$ and v be a point uniformly distributed in $B(0,2R)\backslash B(0,R)$.

$$\begin{aligned} \mathbb{E}\left[d_0^+(u)\right] &= \lambda \int_0^{2\pi}\int_0^R (\pi R^2 - A(r))rdrd\theta \\ &= \lambda R^2 \frac{3\sqrt{3}}{4} \end{aligned}$$

$$\mathbb{E}\left[d_0^-(v)\right] = \lambda \frac{2}{3R^2}\int_R^{2R} A(r)rdr = \lambda R^2 \frac{\sqrt{3}}{4}$$

$$\mathbb{E}\left[d_0^-(v) | v \in N_2\right] = \frac{\mathbb{E}\left[d_0^-(v)\right]}{\mathbb{P}(d_0^-(v) > 0)}$$

$$\mathbb{P}\left(d_0^-(v) > 0\right) = 1 - \frac{2}{3R^2}\int_R^{2R} exp\{-\lambda A(r)\}rdr$$

$$\mathbb{E}\left[|N|\right] = \lambda\pi R^2$$

$$\begin{aligned} \mathbb{E}\left[|N_2|\right] &= 3\lambda\pi R^2 \mathbb{P}\left(d_0^-(v) > 0\right) \\ &= 3\lambda\pi R^2\left(1 - \frac{2}{3R^2}\int_R^{2R} exp\{-\lambda A(r)\}rdr\right) \end{aligned}$$

All these quantities can be computed in the same way. We use the following properties of a Poisson point process: conditioned by the number of points in $B(0,R)$ (resp. in $B(0,2R)\backslash B(0,R)$), the points are independently and uniformly distributed in $B(0,R)$ (resp. in $B(0,2R)\backslash B(0,R)$) and are independent of the points of $B(0,2R)\backslash B(0,R)$ (resp. $B(0,R)$). For instance, we are able to find the distribution of the number of points of N_2 covered by a point u of $B(0,R)$: it is a discrete Poisson law of parameter $\lambda\nu(B(u,R)\backslash B(0,R))$ (ν is the Lebesgues measure in $I\!R^2$) with u uniformly distributed in $B(0,R)$.

B. Analysis of the first step of the MPR selection

In this section, we compute several quantities relative to the first step of the algorithm. We use MPR_1 to denote the set of points of N which are selected as MPR during the first step of the algorithm. In the next proposition, we give the mean number of points $v \in N_2$ such that $d_0^-(v) = 1$. These points are called isolated points in the remaining of the paper. The points of N, neighbors of these isolated points, belong necessarily to MPR_1 as they are the only way to reach them from node 0. However, this quantity does not give the size of MPR_1, since several isolated points can be reached by the same MPR_1 point.

***Proposition** 2:* Let v be a point uniformly distributed in $B(0,2R)\backslash B(0,R)$ and D the set of points v such that $d_0^-(v) = 1$.

$$\mathbb{P}\left(d_0^-(v) = 1\right) = \frac{2}{3R^2}\int_R^{2R} \lambda A(r) exp\{-\lambda A(r)\}rdr$$

$$\mathbb{P}\left(d_0^-(v) = 1 | v \in N_2\right) = \frac{\mathbb{P}\left(d_0^-(v) = 1\right)}{\mathbb{P}\left(d_0^-(v) > 0\right)}$$

and,

$$\mathbb{E}\left[|D|\right] = 2\pi\lambda^2 \int_R^{2R} A(r) exp\{-\lambda A(r)\}rdr$$

In the next proposition we give a lower and an upper bound for the mean size of MPR_1.

***Proposition** 3:* Let u be a point uniformly distributed in $B(0,R)$.

$$\mathbb{P}\left(u \in MPR_1\right) \geq \frac{2}{R^2}\mathbb{P}\left(d_0^+(u) > 0\right) \times$$

$$\int_0^R \int_R^{R+r} f(x,r,R) \exp\left\{-\lambda\left(2\pi R^2 - A_1(R,x,R)\right)\right\} rdxdr$$

with

$$f(x,r,R) = -\frac{\lambda}{1-\exp\{-\lambda(A_1(R,r,R)-\pi R^2)\}} \times \left[\frac{\partial}{\partial x}A_1(x,r,R) - 2\pi x\right] \exp\{-\lambda\left(A_1(x,r,R)-\pi x^2\right)\}$$

The next formula is the direct consequence of the formula above,

$$\mathbb{E}[|MPR_1|] \geq 2\lambda\pi\mathbb{P}\left(d_0^+(u)>0\right)\int_0^R\int_R^{R+r} f(x,r,R) \times \exp\{-\lambda\left(2\pi R^2 - A_1(R,x,R)\right)\} r dx dr$$

Moreover, since there is at least one isolated point by point of MPR_1, the mean number of isolated points offers an upper bound:

$$\mathbb{E}[|MPR_1|] \leq \mathbb{E}[|D|]$$

Proof: We just give here a sketch of the proof. We obtain a bound on the probability that a point in N belongs to MPR_1. A sufficient condition that $u \in MPR_1$ is that the farthest point of $N(u)$ from 0, denoted w, is such that $d_0^-(w) = 1$. Given the distance of u from 0 (expressed by r in the formula), we calculate the probabilistic distribution function of the distance of w and deduce the density function of the distance between w and 0 (denoted $f(x,r,R)$ in the formula). Given the distance between w and u, we are able to compute the probability that $d_0^-(w) = 1$.

This bound is very accurate since, in most cases, the isolated points are the farthest points from 0. ■

We are also interested in the spatial distribution of the MPR_1 points. For u, a neighbor of 0 at distance r ($r \leq R$), we give a lower and an upper bound on the probability that u belongs to MPR_1.

Proposition 4: Let u be a point at distance r ($r \leq R$) from the origin.

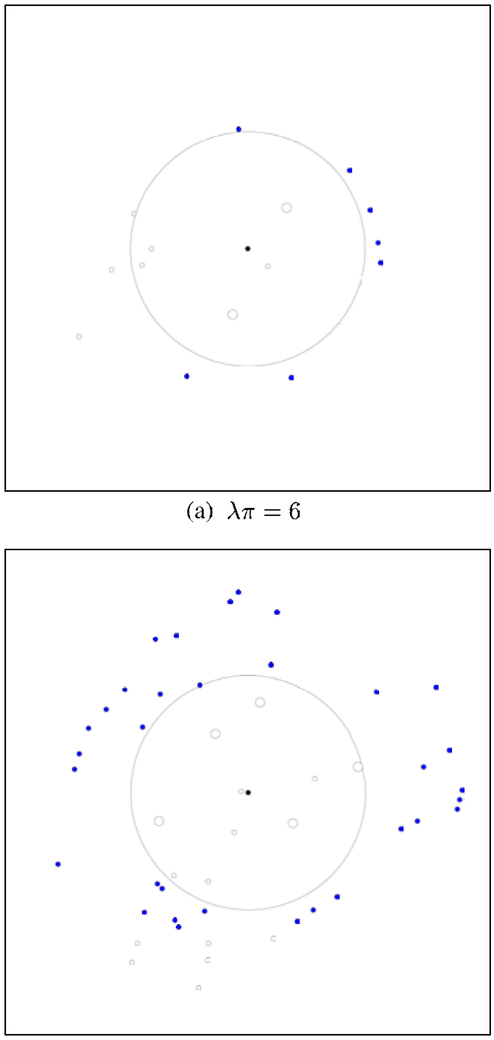
$$\mathbb{P}(u \in MPR_1) \geq \left(1-\exp\{-\lambda(\pi R^2 - A(r))\}\right) \times \int_R^{R+r} f(v,r,R)\exp\{-\lambda(2\pi R^2 - A_1(R,v,R))\}dv$$

$$\mathbb{P}(u \in MPR_1) \leq 1 - \left(1-\exp\{-\lambda\frac{A(R+r)}{2}\}\right)^2$$

Proof: The lower bound is obtained in the same way as the bound in Proposition 3 but given the distance between the origin and its neighbor u. The upper bound is obtained as follows. If there is a point in the lower and upper semi-intersections between the two circles as illustrated in Figure 4(a), the point u does not cover any isolated point. In fact, all the neighbors of u in N_2 are covered by u and by at least one point of the two semi-intersections. This gives a lower bound on the probability that u does not belong to MPR_1 from which we deduce the upper bound on the probability that u belongs to MPR_1.

■

IV. Numerical Results and Simulations

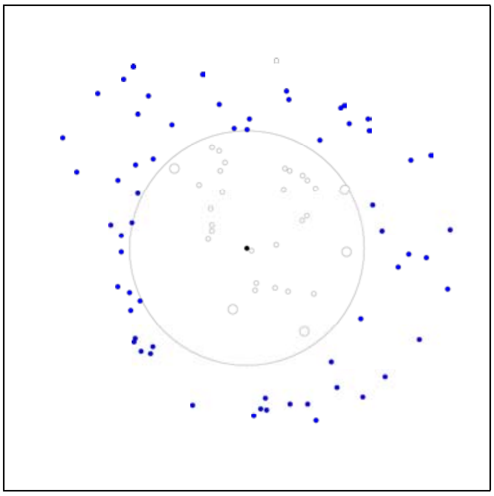


(a) $\lambda\pi = 6$

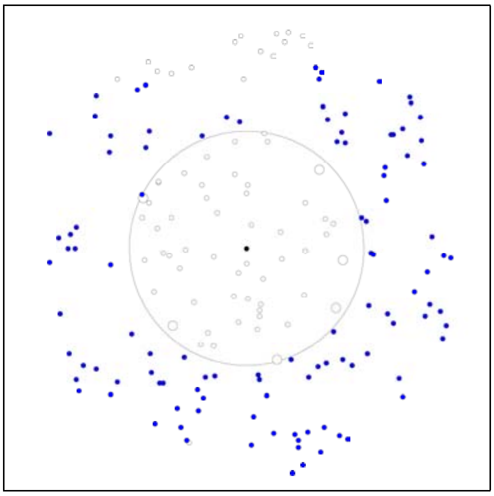
(b) $\lambda\pi = 15$

Fig. 1. MPR selection with $\lambda\pi = 6$ and $\lambda\pi = 15$.

In simulations, the nodes of the network are represented by a Poisson point process in $B(0,2)$ ($R = 1$) of intensity $\lambda > 0$. We add a point at 0. We study for this point the number of MPR selected at each step of the MPR selection and we show that the analytical results are very close to the simulations' ones. In Figures 1(a), 1(b), 2(a) and 2(b), we have represented samples of the model for different values of $\lambda\pi$. We



(a) $\lambda\pi = 30$

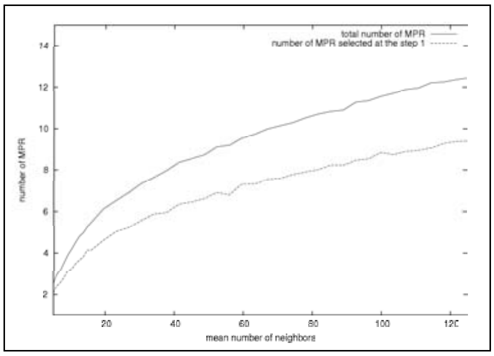


(b) $\lambda\pi = 45$

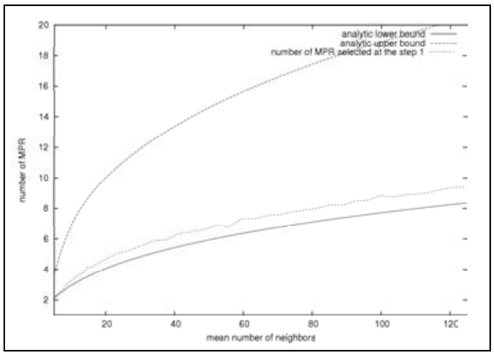
Fig. 2. MPR selection with $\lambda\pi = 30$ and $\lambda\pi = 45$.

point out that $\lambda\pi$ is the mean number of neighbors of a node in the network. The point at the origin for which we compute the MPR is the black point in the middle of the figures. The points in the central circle represent the set N (the neighbors of 0). The larger points of this set represent the MPR_1 points (points selected as MPR during the first step of the algorithm). The points outside the circle are the points of N_2 (the 2-neighborhood of 0) and the blue points are the points of N_2 which are covered by the MPR_1 points.

We note that in all four cases almost the entire 2-neighborhood of 0 is covered by the MPR_1 set. The addition of one MPR would suffice to cover the rest of N_2. We have shown in the previous section that there is an appreciable number of isolated points giving rise to a certain number of MPR_1 points. These MPR_1 points seem to be distributed very close to the boundary of $B(0, R)$ and regularly scattered on it (which confirms the results of the Proposition 4). Therefore, they cover a very large part of N_2.



(a) Mean number of MPR and MPR_1 obtained by simulation when $\lambda\pi$ varies.

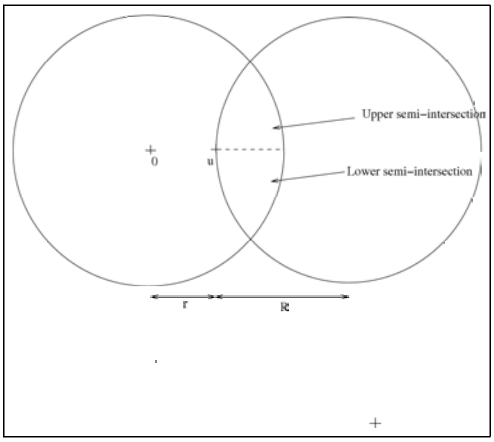


(b) Comparison of the number of MPR_1 obtained with simulations and the analytical bounds when $\lambda\pi$ varies.

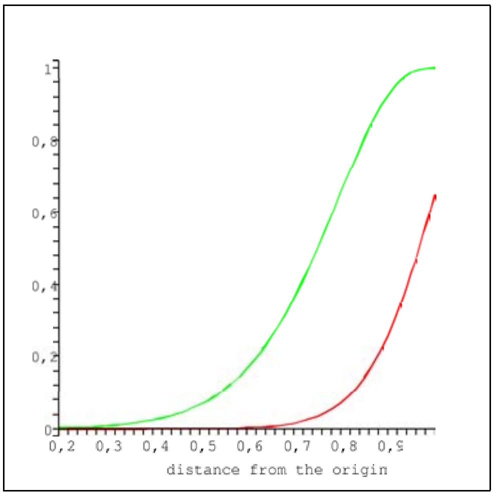
Fig. 3. Mean number of MPR and MPR_1 obtained by simulation and comparison with the lower bound.

Figure 3(a) shows the mean number of MPR and MPR_1 obtained by simulation. We observe that approximately 75% of the MPR are in MPR_1 which confirms that the MPR_1 cover almost the whole 2-neighborood. In Figure 3(b), we have plotted the mean number of MPR_1 obtained by simulation and the analytic lower bound. As explained before, the lower bound is very close to the mean size of the set MPR_1.

The lower and upper bounds on the probability that a point belongs to MPR_1 described in Proposition 4 allow us to show that the MPR_1 points are very close to the boundary. In Figure 4(b), these bounds are plotted when the distance between 0 and its neighbors varies from 0.2 to 0.999 and with $\lambda = 15$. These curves incontestably show that MPR_1 points are distributed closely to the boundary of $B(0, 1)$. We point out that these results depend on λ: as λ increases, the distance



(a) The two semi-intersections used in the proof of Proposition 4.



(b) Lower and upper bounds on the probability of belonging to MPR_1 *w.r.t.* the distance from the origin.

Fig. 4. Semi-intersections used in the proof of Proposition 4 and the bounds on the probability of belonging to MPR_1 *w.r.t.* the distance from 0.

between MPR_1 points and 0 increases too. As described in [6], for messages broadcasted over the network, part of the redundancy perceived by nodes is linked to the size of the intersection between the MPR radio areas. Since the distance between a point and its MPR is great, these intersections are minimal, thus minimizing the redundancy.

V. Conclusion

In this paper, we have computed some quantities relative to the MPR selection algorithm in OLSR. We have shown that approximately 75% of the MPR are chosen during the first step of the algorithm. Since this step is always necessary for the MPR set to cover the whole 2-neighborhood, variants of the algorithm used in OLSR lead to similar performances. We have also highlighted the fact that these MPR are distributed close to the radio range boundaries, limiting the overlap between MPR. This feature also underlines a robustness problem. Indeed, if 75% of node u's MPR cover at least one isolated node in $N_2(u)$ and if some $MPR(u)$ fail, there is a great probability that at least one node in $N_2(u)$ does not receive messages from u.

These results have been presented for a particular model using Poisson point process. Other models, more realistic, which take into account the properties of the radio layer could be considered in future works. Results obtained here could be compared to simulations considering CDMA network or 802.11 network.

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